To protect your own network & to destroy your opponent's:

The importance of being explosive

Coventry, April 7, 2016

Outline:

- Explosive percolation [D. Achlioptas et al., Science **323**, 1453 (2009)]
- Network immunization & targeted network attack
- Connecting both:
  Explosive immunization [P. Clusella et al., arXiv:1604.00073 (2016)]

Explosive Percolation

Ordinary bond percolation:

- Graph $G = (V, L)$
  $V = \{1, \ldots N\}$: vertices
  $L \in V \times V$ set of *possible* bonds (i, k)

- Start with all vertices present, but none of the bonds
- Choose a new random bond:
  choose a random pair (i, k) of vertices for which a bond is possible,
  but which is not yet connected
- Add this bond
- Repeat

When number of added bonds is $O(N)$: appearence of large cluster

Explosive bond percolation:

Bonds are not added reandomly, but:

- Choose two random "candidate bonds" (i, k) and (j, m)
- From these candidates, select the one that leads to **smaller** cluster
- Add this bond

Trivial variations:

- Site instead of bond percolation:
  choose random empty candidate *sites*
- > 2 candidates
- Different "score": instead of new size = sum of old sizes as score, take e.g.:
  - product of old sizes ("product rule")
  - sum of square roots of old sizes
  - in site percolation: degree of candidate

Common to all variants:

Formation of large cluster is delayed,
until all "harmless" sites / bonds are used up

When finally non-harmeless sites / bonds have to be inserted:

$\rightarrow$ **largest cluster grows explosively**

Achlioptas et al.:

→ **first example of discontinuous phase transition in percolation !!**

Doubly wrong:

- e.g. bootstrap percolation is discontinuous
- explosive percolation is not (O. Riordan & L. Warnke 2011, R.A. da Costa et al. 2010))

But: EP has very unusual finite size scaling (PG, C. Christensen, G. Bizhani, S.-W Son, M. Paczuski, 2011)

Why was Achlioptas et al. so much cited (320 times!)?

EP looks very unphysical

Example for **“power of choice”** (Y. Azar 1994):

Changing an optimization strategy from blind to greedy can improve it dramatically
even if “greedy” means choosing from few candidates

Other example:
Uniformity of lattice covering by random versus self-repelling walkers
PG & H. Freund, Physica A 1993;
C. Avin & B Krishnamachari 2008

.

But who needs a percolation cluster that grows as slow as possible?

Network immunization

- Very serious veneral disease runs in population of 10 million
- Vaccine exists, that can be given even after infection, ...
- ..., but there are only 1 million units
- Who is to vaccinated?
- Lady comes to doctor:
  "please vaccinate me, I have been raped by Mr. X"
- Doctor: "go home; I will vaccinate X"

To optimize the effect of limited vaccination capability,
$\rightarrow$ vaccinate most dangerous spreaders!!

First approximation:
most dangerous = highest degree (most contacts)

But not always!!
Assume node X is part of a clique:
Then vaccinating X is useless,
because anybody who infected X also has infected the entire clique

Optimal network immunization is NP complete
A. Braunstein et al., arXiv:1603.08883

most important spreaders
$\neq$ most important blockers (i.e. to be vaccinated)

(in contrast to claims in F. Morone et al. 2015)

Most influential nodes determined by “page rank” (Google) are not good blockers

Network immunization & targeted network destruction

Immunization of network against a bug by node vaccination
= destruction of the bug's network

$\rightarrow$ **network immunization $\equiv$ targeted destruction**

It just depends on the point of view

Direct strategy:

Vaccinate dangerous ones first!

Inverse strategy:
(C.M. Schneider et al., Europhys. Lett. 2012)

Determine first those NOT to be vaccinated!

Success measure:
$S(q)$ = relative size of largest connected cluster, when a faction q of nodes are vaccinated

If
non-vaccinated node ≡ established node
vaccinated node ≡ removed node

→ **inverse strategy ≡ Achlioptas' problem**

Improvements over Schneider et al.:

- Choose "harmless" nodes from finite set of candidates (ca. 10 -1000)
- Use fast Newman-Ziff algorithm for node insertion
- Use better scores
  - Start at $q = 1, S(q) = 1/N$
  - as long as $S(q) < 1/\sqrt{N}$, use score $\sigma^{(1)}$ which is large
    - if large cluster is created
    - if degree of established ("de-vaccinated") node is large
    - if established node havs not many too strong hubs (they will be vaccinated anyhow)
  - after giant cluster is formed, switch to score $\sigma^{(2)}$ which
    - no longer looks at largest cluster
    - prevents depletion of intermediate size clusters

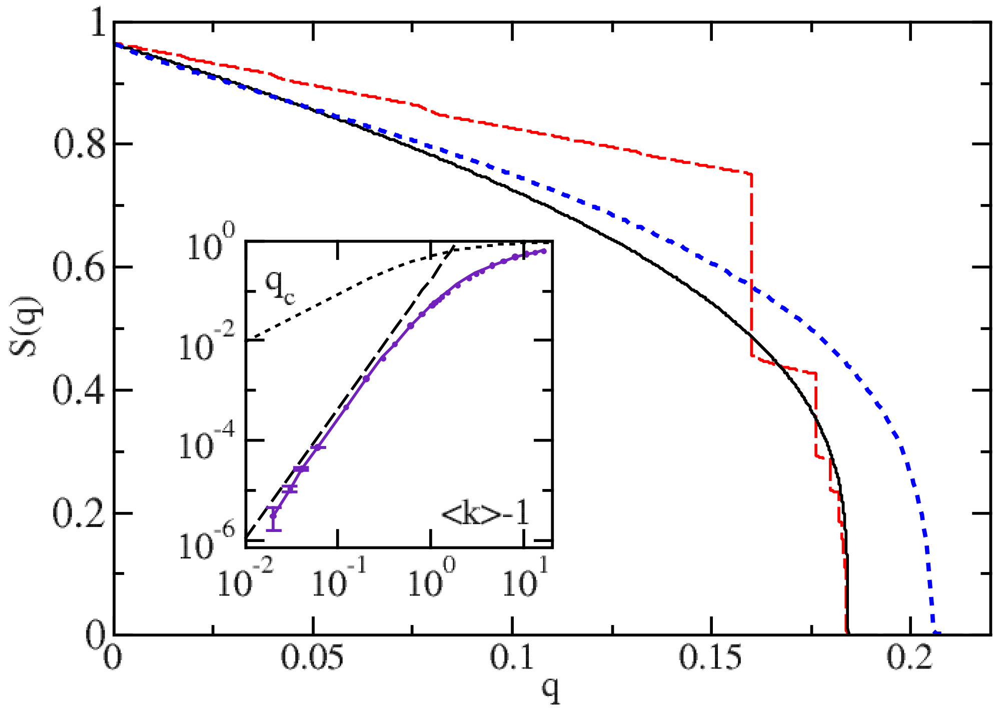

Figure 1: Erdös-Renyi network, $\langle k \rangle = 3.5, N = 10^6$.
Blue dashed line: F. Morone & H. Makse;
Black curve: $\sigma^{(1)}$ & $\sigma^{(2)}$;
Red curve: $\sigma^{(1)}$ only.

→ **Best heuristic up to March 2016:**

- complexity $O(N)$ up to log's
- smallest q_c (= point where giant cluster appears)
- smallest $S(q)$ for $q < q_c$

Better algorithms for q_c:
S. Mugisha & H.-J. Zhou, arXiv:1603.05781
A. Braunstein et al., arXiv:1603.08883

Both use belief propagation;

Both use the fact that critical q for cluster percolation is,
for $N \to \infty$,
also the critical point for forming loops

Strategy:

remove first all loops by node vaccinations,
then it is easy to disect the resulting tree

$\to$ very large (bad) $S(q)$ for $q < q_c$.

Summary:

Strategy based on explosive site percolation concept gives excellent network immunization;

Similar concepts (but based on explosive bond percolation) should work for quarantaine & community detection